

# Quantum Brownian motion and a theorem on fundamental $1/f$ noise

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**Abstract.** We consider quantum Hamiltonian systems composed of mutually interacting “dynamical subsystem” with one or several degrees of freedom and “thermostat” with arbitrary many degrees of freedom, under assumptions that the interaction ensures irreversible behavior of the dynamical subsystem, that is finite diffusivities of its coordinates in thermodynamically equilibrium state and finite drift velocities and mobilities in non-equilibrium steady state in presence of external driving forces. It is shown that, nevertheless, regardless of characteristics of the interaction, the diffusivity and mobility have no certain values but instead vary from one observation to another and undergo  $1/f$ -type or flicker-type low-frequency fluctuations.

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## 1. Introduction

Nearly 30 years ago in [1, 2] and then in [3, 4, 5, 6] an original explanation of electronic  $1/f$ -noise was suggested by Prof. G.N. Bochkov and me and putted in phenomenological statistical model which easy produces correct estimates of the noise level and directly connects to rigorous statistical mechanics (for later looks at this model see [7, 8]). Key idea of our explanation was that own (thermal) fluctuations in rates of various transport processes in many-particle systems do not cause a “back reaction” and therefore have no definite “relaxation times”. Hence, that are scaleless fluctuations, with power-law low-frequency spectrum (which thus has no relation to folklore “composition of lorentzians”).

In [1, 2, 3, 4, 5, 6] this principal idea was exploited in specific terms of electric charge transport and “Brownian motion” of charge carriers (one may see also [9, 10, 11, 12]). But, clearly, it can be applied also to different processes in large variety of physical systems (in most wide sense of the word “physical” [7]). Particularly, to Brownian

motion (“random walks”) of a fluid particles [13, 14, 15, 16, 17, 18, 20], as well as to collective fluctuations and non-equilibrium processes in fluids [15, 16] and solids [21].

Simultaneously, the works [13, 21] and later [14, 15, 16, 17, 18] and [11], in fact, presented substantiations of the mentioned key idea on the base of rigorous statistical mechanics, and confirmed the sentence from [5] that “ $1/f$ -noise is a kind of tribute to be paid to the dynamics for dissipation and irreversibility properties of physical systems” (my translation from Russian original).

This sentence fully agrees with results of the deep N.Krylov’s investigation [22] (discovered by me when preparing [13]) where it was shown that in many-particle statistical mechanics, firstly, relative frequencies of observable events can be different at different system’s phase trajectories (experiments), regardless of duration of observations and time averaging (see characteristic mental example in introductions in [16] and [14] and more examples in [7]). Therefore, generally one has no rights to presume any *a priori* certain (even let unknown numerically) “probabilities” of events. Secondly (as the consequence), it is possible that statistical correlations do exist even between events what are independent in physical (cause-and-consequence) sense.

In [13, 21, 14, 15, 16] I demonstrated that real “fundamental”  $1/f$ -noise (i.e. such  $1/f$ -noise whose spectrum has no saturation at zero frequency) is just manifestation of the Krylov’s uncertainty of events’ probabilities (additional explanations can be found in [7] and introductory or discussion-resume sections in [8, 9, 17, 18, 19] and [11]).

Unfortunately, the enumerated works, - including the Krylov’s book, - yet has not excited any scientific-community’s response except neglect or aversion ‡ . Interestingly, such aversion was pointed out and commented already by Krylov himself [22], as product of common prejudices educated by standard “probability-theoretical” view at physical world and its statistical description § . Hence, we have to continue activity in this field as far as possible.

In this paper our aim is to illustrate the aforesaid ideology (and once again confirm its correctness) by example of simple Hamiltonian microscopic model of quantum Brownian motion. More concretely, to show that if an interaction of (quantum) particle with (quantum) thermostat make this particle “Brownian”, that is ensures diffusive character of its chaotic motion, - when mean square of its displacement grows

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Thus showing that progress in fundamental science also undergoes fundamental  $1/f$ -type fluctuations. For example, manuscripts [15] were rejected by JSTAT and JSP, respectively, [16] by Physica A, [17] by JSP, EPL and JMP, [8] and [10]-[11] by JETP, [19] by PRE, JPA and JSP, etc., in all cases without meaningful or even any motivation.

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The modern notion of “dynamical chaos” still has not changed this situation, since, instead of investigating specificity of the chaos in (infinitely-) many-particle Hamiltonian systems, - where it naturally creates  $1/f$ -noise [16, 21] and thus “kinetic non-ergodicity” [7, 16], - scientists try to squeeze it in “Procrustean bed” formed with “Markovian partitions”, “Bernoullian flows” and primitive stochastic processes with *a priori* introduced “probabilities of elementary events”. Although there is also concept of such “K-flows” what can not be re-coded to Bernoullian flows [23].

proportionally to time, - then such interaction ensures also non-zero  $1/f$ -type fluctuations of its diffusivity (and hence mobility). In other words, the latter has no certain value but varies from one measurement to another.

## 2. A class of systems under consideration

Let us consider a quantum system with Hamiltonian

$$H = H_d(q, p) + H_b(q) , \quad (1)$$

$$H_d(q, p) = \frac{p^2}{2m} - f \cdot q , \quad (2)$$

where  $H_d(q, p)$  represents “dynamical subsystem” with canonical variables (coordinates and momenta)  $q$  and  $p$ , whose operators satisfy standard commutation relations

$$[q_\alpha, p_\beta] = i\hbar \delta_{\alpha\beta} , \quad (3)$$

and  $H_b(q)$  represents “thermal bath” (“thermostat”) along with interaction between it and the dynamical subsystem (DS).

Our first principal assumption is that our system is “translationally invariant” in respect to the coordinates  $q$ , in the sense that properties of the operator  $H_b(q)$ , - as considered in the thermostat’s Hilbert (phase) space, - do not depend on parameters  $q$ . Formally, this means that there exist such Hermitian operators  $\Pi$ , - defined in the thermostat’s space, - that

$$H_b(q + a) = \exp(-ia\Pi/\hbar) H_b(q) \exp(ia\Pi/\hbar) \quad (4)$$

In other words, from thermodynamical point of view the thermostat is indifferent to  $q$ ’s values.

Under such assumption, one can treat variables (observables)  $q$  and  $v = p/m$  like coordinate and velocity of unrestricted (generally multi-dimensional) “Brownian motion”.

We want to consider possible statistical characteristics of these motion, at that basing, of course, on the von Neumann (quantum Liouville) evolution equation for full system’s density matrix  $\rho$ , that is

$$\dot{\rho} = [H, \rho]/i\hbar \equiv \mathcal{L} \rho , \quad (5)$$

with  $[A, B] \equiv AB - BA$  and  $\mathcal{L}$  being the Liouville super-operator.

## 3. Quasi-classic representation

It is convenient to consider Eq.5 in the form most closely unifying quantum and classical treatments of DS’s variables, that is in quasi-classical or Wigner representation. Simultaneously, by reasons what will be clear later, we want to go from the density matrix,  $\rho$ , to an equivalent characteristic function of variables of our system. We will make this in three steps.

1. Let  $|q\rangle$  be eigen-vectors of operators  $q$ , and let us introduce functions-operators

$$\rho(t, x, y) = \langle x + y/2 | \rho | x - y/2 \rangle , \quad (6)$$

$$\rho(t, x, p) = \int \exp(-ipy/\hbar) \rho(t, x, y) d^d y / (2\pi\hbar)^d , \quad (7)$$

where  $d$  is number of pairs  $\{q, p\}$ , i.e. degrees of freedom of the dynamical subsystem (DS). The function-operator  $\rho(t, x, p)$  is jointly the Wigner's probability distribution function of DS's variables, - with  $x$  representing the coordinates, - and density matrix of the thermostat.

From viewpoint of this distribution, evidently,

$$\rho(t, x, y) = \int \exp(ipy/\hbar) \rho(t, x, p) d^d p \quad (8)$$

is nothing but characteristic function of the momenta  $p$ , in the same sense of the words “characteristic function” (CF) as in the probability theory. Or, to be more precise,  $\rho(t, x, y)$  is a hybrid of CF, - in respect to DS's momenta, - and probability distribution (PD), or density matrix, in respect to DS's coordinates and all thermostat's variables.

2. Next, let us make transform

$$\rho(t, x, y) = \exp(-ix\Pi/\hbar) R(t, x, y) \exp(ix\Pi/\hbar) , \quad (9)$$

$$\rho(t, x, p) = \exp(-ix\Pi/\hbar) R(t, x, p) \exp(ix\Pi/\hbar)$$

After it, in terms of new (distribution) function-operator (density matrix)  $R(t, x, y)$ , Eq.5 takes form

$$\dot{R} = \frac{i\hbar}{m} \frac{\partial^2 R}{\partial x \partial y} + \mathcal{L}_f R = \mathcal{L} R , \quad (10)$$

$$\mathcal{L}_f R \equiv \frac{i}{\hbar} f \cdot y R + \frac{1}{m} \frac{\partial}{\partial y} [\Pi, R] + \frac{1}{i\hbar} \{ H_b(y/2) R - R H_b(-y/2) \} \quad (11)$$

Clearly, one can say that  $R$  describes thermostat in movable frame connected to DS coordinates  $q$  (via relative coordinates, if say in classical language).

One more Fourier transform, in addition to (8), in respect to  $x$ , produces function-operator

$$R(t, k, y) = \int \exp(ikx) R(t, x, y) d^d x , \quad (12)$$

which is joint semiclassical CF of all DS's variables “hybridized” with thermostat's density matrix. The corresponding evolution equation directly follows from Eq.10:

$$\dot{R} = \frac{\hbar k}{m} \frac{\partial R}{\partial y} + \mathcal{L}_f R \equiv \mathcal{L} R \quad (13)$$

3. For last step, we assume that, - as usually in many-particle physics (see e.g. [24]), - all possible states of the thermostat can be described as results of actions of definite particles (or quasi-particles) and/or quanta creation and annihilation operators,  $c_s^\dagger$  and  $c_s$ , onto “vacuum state”. Index “s” here enumerates various sorts and modes of particles and quanta. Then all thermostat's observables, including  $H_b(q)$  and  $\Pi$ , can be treated as functions of  $c_s^\dagger$  and  $c_s$ . Correspondingly, thermostat's component of

full density matrix of our system can be completely described in terms of quantum CF defined e.g. like  $\text{Tr}_B \exp(z_s^* c_s) \rho \exp(z_s c_s^\dagger)$ , where  $\text{Tr}_B$  denotes trace over thermostat space, the repeated indices “s” mean summation (or/and integration), and  $z_s$  and  $z_s^*$  are test, or probe, parameters, either complex  $c$ -numbers or Grassmann numbers, depending on whether  $c_s^\dagger$ ,  $c_s$  are Bose or Fermi operators (at that, generally,  $z_s$  and  $z_s^*$  are thought as independent, not mutually conjugated, variables).

Then, analogously, let us introduce full CF of our system by

$$\mathcal{F}\{t, k, y, z, z^*\} = \text{Tr}_B \exp(z_s^* c_s) R(t, x, y) \exp(z_s c_s^\dagger) \quad (14)$$

Importantly, under the coherent-state representation of  $R$  such defined CF coincides with mere CF of corresponding “quasi-probability density”. Therefore  $\mathcal{F}\{t, k, y, z, z^*\}$  is fully semi-classical object, and we can speak about it applying usual “probability-theoretical” terminology.

In order to write out an evolution equation for it, first for any operator  $A$ , - composed of various  $c_s^\dagger$  and  $c_s$ , - let us introduce two operators  $\mathcal{A}^+$  and  $\mathcal{A}^-$  acting in space of functions of the probe parameters and defined as follow:

$$\begin{aligned} \exp(z_s c_s^\dagger) \exp(z_s^* c_s) A &= \\ &= \mathcal{A}^+ \left( z, z^*, \frac{\partial}{\partial z}, \frac{\partial}{\partial z^*} \right) \exp(z_s c_s^\dagger) \exp(z_s^* c_s) , \end{aligned} \quad (15)$$

$$\begin{aligned} A \exp(z_s c_s^\dagger) \exp(z_s^* c_s) &= \\ &= \mathcal{A}^- \left( z, z^*, \frac{\partial}{\partial z}, \frac{\partial}{\partial z^*} \right) \exp(z_s c_s^\dagger) \exp(z_s^* c_s) \end{aligned} \quad (16)$$

Thus  $\mathcal{A}^+$  and  $\mathcal{A}^-$  are composed of multiplications and differentiations in respect to probe parameters. Obviously, structure of  $\mathcal{A}^+$  and  $\mathcal{A}^-$  is unambiguously determined by that of  $A$  and commutation rules of  $c^\dagger$ ’s and  $c$ ’s. At that, anyway

$$\left[ \mathcal{A}^+ \left( z, z^*, \frac{\partial}{\partial z}, \frac{\partial}{\partial z^*} \right) - \mathcal{A}^- \left( z, z^*, \frac{\partial}{\partial z}, \frac{\partial}{\partial z^*} \right) \right]_{z=z^*=0} = 0 \quad (17)$$

Exploiting these definitions, one easy transforms Eq.13 into equation for the full CF  $\mathcal{F}\{t, k, y, z, z^*\}$ :

$$\dot{\mathcal{F}} = \left[ \frac{\hbar k}{m} \frac{\partial}{\partial y} + \mathcal{L}_f \right] \mathcal{F} = \mathcal{L} \mathcal{F} , \quad (18)$$

$$\mathcal{L}_f = \frac{i f \cdot y}{\hbar} + \frac{1}{m} \frac{\partial}{\partial y} [\mathcal{P}^+ - \mathcal{P}^-] + \frac{1}{i\hbar} [\mathcal{H}_b^+(y/2) - \mathcal{H}_b^-(-y/2)] \quad (19)$$

Here  $\mathcal{H}_b^\pm(q)$  and  $\mathcal{P}^\pm$  are the operators produced, - according to Eqs.15-16, - from  $H_b(q)$  and  $\Pi$ , respectively, and their arguments are omitted for brevity.

Further, let us discuss construction and properties of solutions of Eq.18 and equivalent Eqs.10 and 13.

#### 4. Equilibrium and steady non-equilibrium states

First, consider stationary solutions of Eq.18, 13, 10 and 5. Because of the thermostat’s property (4) we can expect that stationary solutions of Eq.10 are independent on  $x$ ,

i.e. “spatially uniform”, if one interprets the DS as “Brownian particle” (BP), while the thermostat as a medium where BP moves (then property (4) means thermostat’s spatial infiniteness and uniformity). Correspondingly, Eqs.13 and 18 have (non-trivial) stationary solutions only when the parameter  $k$  (“wave vector”) equals to zero,  $k = 0$ . Denoting them by  $R_{st}(y; f)$  and  $\mathcal{F}_{st} = \mathcal{F}_{st}\{y, z, z^*; f\}$ , we can write

$$\mathcal{L}_f R_{st} = 0 \quad , \quad \mathcal{L}_f \mathcal{F}_{st} = 0 \quad , \quad (20)$$

with  $\mathcal{L}_f$  defined by (11) and (19), respectively.

Among all possible solutions of this equation, most physically interesting are that characterized by definite thermostat’s temperature. In absence of the “external driving force”, i.e. at  $f = 0$ , such solution is thermodynamically equilibrium one determined by

$$R_{eq}(y) = \rho_{eq}(y) \propto \langle y/2 | \exp \{ -[p^2/2m + H_b(q)]/T \} | -y/2 \rangle \quad , \quad (21)$$

$$\mathcal{F}_{eq}\{y, z, z^*\} = \text{Tr}_B \exp(z_s^* c_s) R_{eq}(y) \exp(z_s c_s^\dagger) \quad , \quad (22)$$

along with obvious normalization condition  $\text{Tr}_B R_{eq}(0) = 1$ ,  $\mathcal{F}_{eq}\{0, 0, 0\} = 1$ .

Further, let us take these equilibrium expressions to be initial conditions to Eqs.13 and 18 with  $f = \text{const} \neq 0$  and  $k = 0$ . Then stationary asymptotic of their solutions at  $t \rightarrow \infty$  will give us such non-equilibrium solution of Eq.20 which can be treated as perturbation of  $R_{eq}$  and  $\mathcal{F}_{eq}$  and therefore also is characterized by definite thermostat temperature.

## 5. Characteristic functional, statistical correlations, cumulants, and statistics of the Brownian motion

Of course, we just have made second principal assumption, namely, that construction of the Hamiltonian  $H_b(q)$  ensures existence of the mentioned stationary asymptotic, at least at sufficiently small finite  $|f|$ .

If it is so, then we can investigate stationary random walk of the BP, - i.e. random changes of the DS variable  $q(t)$ , - by considering non-stationary solutions of Eqs.10 or 13 or 18 at  $t > 0$  with initial conditions

$$\begin{aligned} R(t=0, x, y) &= \delta(x) R_{st}(y; f) \quad , \quad R(t=0, k, y) = R_{st}(y; f) \quad , \\ \mathcal{F}\{t=0, k, y, z, z^*\} &= \mathcal{F}_{st}\{y, z, z^*; f\} \quad , \end{aligned} \quad (23)$$

respectively. Evidently, these initial conditions are quasi-classical representations of such state of DS (BP) which is balanced in respect to its interaction with thermostat but, at the same time, localized in the  $q$ -space at  $q(t=0) = 0$ . Therefore solution of Eq.10 for  $t > 0$  determines probability density distribution,  $W(t, x)$ , of  $x(t) = q(t) - q(0)$  (BP’s path, or displacement, during time  $t$ ), while Eqs.13 and 18 characteristic function of this distribution:

$$\begin{aligned} W(t, x) &= \text{Tr}_B R(t, x, y=0) = \langle \delta(x(t) - x) \rangle \quad , \\ W(t, k) &= \int e^{ikx} W(t, x) d^d x = \end{aligned} \quad (24)$$

$$= \text{Tr}_B R(t, k, y = 0) = \mathcal{F}\{t, k, y = 0, z = 0, z^* = 0\} = \langle e^{ikx(t)} \rangle \quad (25)$$

We introduced the angle brackets as standard comfortable designation of statistical ensemble averaging. With this designation, in the quasi-classical language, we can write

$$\mathcal{F}\{t, k, y, z, z^*\} = \langle \exp [ikx(t) + i\xi v(t) + z_s c_s^*(t) + z_s^* c_s(t)] \rangle, \quad (26)$$

where  $v(t) = p(t)/m = dx(t)/dt$  represents velocity of  $q(t)$ 's changes, so that

$$x(t) = \int_0^t v(\tau) d\tau, \quad (27)$$

and  $\xi \equiv my/\hbar$ .

Expression (26) visually shows that  $F\{t, k, y, z, z^*\}$  is full characteristic function of all variables of the whole system “DS plus thermostat” (or, to be precise, characteristic functional, since for infinitely large thermostat  $c_s$  and  $c_s^*$  form continuum set of variables). Taking in mind general properties of characteristic functions in the probability theory [25], we can write also

$$F\{t, k, y, z, z^*\} = W(t, k) \Theta\{t, k, y, z, z^*\} \mathcal{F}_{st}\{y, z, z^*; f\}, \quad (28)$$

where the middle multiplier on the right,  $\Theta$ , contains all cross-correlations between the path  $x(t)$ , on one hand, and  $v(t)$ ,  $c_s(t)$  and  $c_s^*(t)$ , on the other hand. This means that

$$\Theta\{t, k = 0, y, z, z^*\} = \Theta\{t, k, y = 0, z = 0, z^* = 0\} = 1 \quad (29)$$

In terms of cumulants, i.e. irreducible correlations, - to be designated by double angle brackets, -

$$\begin{aligned} \ln \Theta\{t, k, y, z, z^*\} &= \sum_{n,l=1}^{\infty} \frac{(ik)^n}{n! l!} \langle\langle x^n(t) \eta^l(t) \rangle\rangle = \\ &= \sum_{n,l=1}^{\infty} \frac{(ik)^n}{n! l!} \int_0^t \dots \int_0^t \langle\langle v(\tau_1) \dots v(\tau_n) \eta^l(t) \rangle\rangle d\tau_1 \dots d\tau_n, \end{aligned} \quad (30)$$

$$\ln W(t, k) = \sum_{n=1}^{\infty} \frac{(ik)^n}{n!} \langle\langle x^n(t) \rangle\rangle, \quad (31)$$

where  $\eta(t) = i\xi v(t) + z_s c_s^*(t) + z_s^* c_s(t)$ .

Before discussion of possible time behavior of the  $x(t)$ 's cumulants in (31) and thus statistics of the Brownian motion, we have to realize most principal property of the evolution operator  $\mathcal{L}$  in Eq.18 determining the  $x(t)$ 's cumulants. Namely, to take into account that

## 6. Spectrum of the Liouville super-operator is purely imaginary

Let  $\Psi_\alpha$  is complete set of mutually orthogonal eigen-vectors of the full system's Hamiltonian (1):  $H|\Psi_\alpha\rangle = E_\alpha|\Psi_\alpha\rangle$ . Then  $\rho_{\alpha\beta} = |\Psi_\alpha\rangle\langle\Psi_\beta|$  is complete set of orthogonal eigen-vectors of the Liouville, or von Neumann, super-operator  $\mathcal{L}$  in (5):

$\mathcal{L}\rho_{\alpha\beta} = i((E_\beta - E_\alpha)/\hbar)\rho_{\alpha\beta}$ . The orthogonality is understood in the sense of the natural “scalar product”  $(A, B) = \text{Tr } A^\dagger B = \text{Tr}_D \text{Tr}_B A^\dagger B$ .

Hence,  $\mathcal{L}$  has purely imaginary spectrum. This statement equally comprises, of course, the evolution operators in Eqs.10, 13 and 18 which are equivalent representations of original Liouville operator. The same can be said about the operators  $\mathcal{L}_f$ .

In other words, these operators can not have real eigen-values or complex ones with nonzero real parts. This trivial truth of statistical mechanics is sufficient ground for fundamental theorem to be claimed in next section.

Before it, notice, first, that in the representation defined by Eqs.9 and 10 the mentioned scalar product reads

$$(A, B) = \int \int \text{Tr}_B A^\dagger(x, -y) B(x, y) d^d y d^d x \quad (32)$$

In the representations producing Eqs.13 and 18, evidently, the variable (“wave vector”)  $k$  in fact plays role of passive parameter of the Liouville super-operator  $\mathcal{L}$ . Therefore its eigen-vectors naturally divide into layers (subsets) which correspond to different  $k$ ’s and can be marked by  $k$  (such treatment of  $\mathcal{L}$ ’s eigen-values will appear below). At that, the scalar product (32) reduces, - in case of Eq.13, - to

$$(A, B) = \int \text{Tr}_B A^\dagger(k, -y) B(k, y) d^d y \quad (33)$$

in each separate layer (an equivalent formula for the case of Eq.18 looks rather cumbersome and will be written out elsewhere).

## 7. Theorem about uncertainty of relaxation, diffusion and dissipation rates

The general hope of conventional “kinetic theory”, or “physical kinetics”, etc., is that a proper interaction of DS (a part of much greater system) with thermostat (the rest of the system) can impel the first of them to irreversible and stochastic behavior characterized by well definite “kinetic coefficients”, relaxation and dissipation rates, etc., at least under the “thermodynamic limit” (for infinitely large thermostat). In respect to presently considered class of systems, this assumption means that all the cumulant functions in (30) and (31) are fast enough decaying (integrable) functions of time differences  $t - \tau_j$  and  $\tau_j - \tau_k$ , respectively. That is all terms of series (30) tend with time to finite limits, so that

$$\Theta\{t, k, y, z, z^*\} \rightarrow \Theta\{\infty, k, y, z, z^*\} \neq \infty, 0, \quad (34)$$

while all terms of series (31) are asymptotically linear time functions:

$$\ln W(t, k) \rightarrow \lambda(ik)t + \text{const}, \quad (35)$$

$$\lambda(ik) = \sum_{n=1}^{\infty} (ik)^n \lambda_n(f)/n! = ik \langle v \rangle + (ik)^2 D + \dots, \quad (36)$$

$$\lambda_n(f) = \lim \langle \langle x^n(t) \rangle \rangle / t, \quad (37)$$



where  $\langle v \rangle = \langle dx(t)/dt \rangle = \langle x(t)/t \rangle$  is mean velocity vector and  $D$  is diffusivity tensor. For “good” enough thermostat and small force  $f$  one expects also that  $\langle v \rangle = \mu f$  with  $\mu$  being mobility tensor.

In terms of the probability theory [25], Eq.35 says that the Brownian path  $x(t)$  behaves as a random process with independent increments and infinitely divisible probability distribution, and Eq.36 specifies that this is diffusive process with asymptotically Gaussian probability distribution.

But, fortunately, these conventional assumptions can not be true. Indeed, if they were true then expressions (34)-(35) as combined with Eqs.18 and 28 would imply that asymptotically

$$\begin{aligned} \lambda(ik) \Theta\{\infty, k, y, z, z^*\} \mathcal{F}_{st}\{y, z, z^*; f\} &= \\ &= \mathcal{L} \Theta\{\infty, k, y, z, z^*\} \mathcal{F}_{st}\{y, z, z^*; f\} , \end{aligned} \quad (38)$$

which would mean that  $\lambda(ik)$  is eigen-value of the Liouville super-operator. But it is certainly impossible since  $\lambda(ik)$  by its definition inevitably has non-zero (negative) real component,  $\Re \lambda(ik) < 0$  at  $k \neq 0$  (while, to be recalled,  $\mathcal{L}$ ’s spectrum is purely imaginary!).

Consequently, the hypothetical linear asymptotic (35)-(37) is wrong, and in fact

$$\frac{\langle\langle x^n(t) \rangle\rangle}{t} \rightarrow \infty \quad (39)$$

for some  $n > 1$  (or  $n > 3$  at  $f = 0$ ), that is all particular increments of the path  $x(t)$  are essentially statistically dependent one on another (correlated with each other) regardless of time distance between them.

At that, values  $n = 1$ , or  $n < 3$  at  $f = 0$ , are excluded from candidates to the super-linearity (as well as to sub-linearity) by our assumptions about existence of steady non-equilibrium state (described by  $\mathcal{F}_{st}$ ) with finite mean (“drift”) velocity  $\langle v \rangle$ , which means finiteness of BP’s diffusivity in equilibrium state (described by  $\mathcal{F}_{eq}$ ) at  $f = 0$  (this follows from the Einstein relation  $D = T\mu$  easy provable for our systems too).

Therefore, the crash of the “independence of increments” implies that generally the super-linear cumulants’ asymptotic (39) takes place at any  $n > 1$  (or  $n > 3$ , if  $f = 0$ ). Then, it remains to realize that this statement is equivalent to statement that mobility and diffusivity possess low-frequency fluctuations like “flicker noise” or  $1/f$ -noise.

In other words, the mobility and diffusivity (as well as related “kinetic characteristics” of  $x(t)$ ’s interaction with thermostat) do not have certain values but change from one experiment (measurement) to another, with a spread what anyway hugely exceeds limits suggested by the “law of large numbers”.

## 8. Conclusion

We have expounded rather general theorem (or, strictly speaking, a “storage” of future rigorous theorem) stating that interaction of one Hamiltonian (sub-) system with another, - let serving as an arbitrarily large thermostat, - never can ensure well certain

quantitative characteristics of irreversible and dissipative behavior of the first of them. Instead, these characteristics undergo significant  $1/f$ -type (“flicker” type) low-frequency fluctuations. Citing once again myself from [5], “ $1/f$ -noise is a kind of tribute to be paid to the dynamics for dissipation and irreversibility properties of physical systems”.

Particular variant of this theorem was obtained in [11]. For better understanding its pre-history, physical meaning and possible applications, see also works referred there and in the Introduction above. Detailed elaboration and concrete applications of our present result will be considered separately.

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